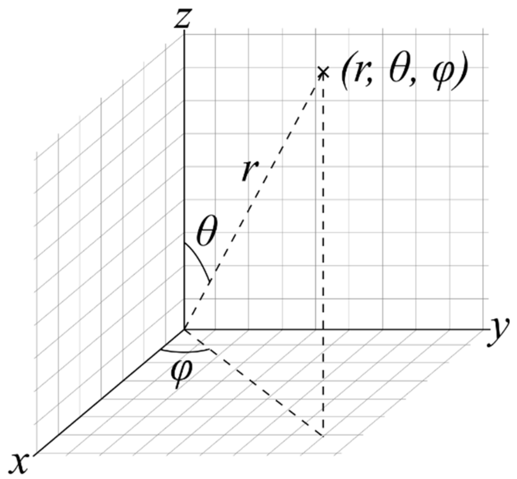
Definition of Coordinate System used in n-Hole Probe Data Reduction.

The spherical coordinate system commonly used in physics (ISO 80000-2:2019 convention) is used throughout.



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = r \begin{pmatrix} \cos(\varphi)\sin(\theta) \\ \sin(\varphi)\sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

The polar angle is θ , the azimuthal angle is φ . Let it be the convention that the probe streamwise axis is aligned with the *x*-axis of the above coordinate system, and that when it is pointing directly into the flow both pitch and yaw angles are zero. This convention means that the yaw angle can be defined as $\beta = \varphi$ but the pitch angle must be defined as $\alpha = 90^\circ - \theta$. After substitution the coordinate system is then defined as follows:

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \|\mathbf{U}\| \begin{pmatrix} \cos(\beta)\cos(\alpha) \\ \sin(\beta)\cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$$

$$\|\mathbf{U}\| = \sqrt{U^2 + V^2 + W^2}$$

U longitudinal velocity component

V lateral velocity component

W vertical velocity component

α pitch angle. Clockwise positive off the xy plane in the direction of the z-axis

β yaw angle. Anti-clockwise positive looking into z-axis

Use in Wind Tunnels:

In wind tunnel use where a right-handed coordinate system *x*-axis is aligned with the flow streamwise direction and the probe is pointing into the flow, the probe is effectively operating as a left-handed coordinate system and a reflection in the XZ plane is required. This results in the negation of *V*.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} U \\ -V \\ W \end{pmatrix}$$

In the case of a wind tunnel right-handed coordinate system that is rotated anti-clockwise by 90° such that the y-axis is the vertical axis rather than the z-axis, one may simply swap the original V and W components (without the negation of *V*). Alternatively, this setup could be considered as a case where the wind tunnel coordinate system is rolled by +90°, which is akin to the probe being rolled by -90°. Applying the appropriate rotation matrix to the probe coordinate system from the previous wind tunnel configuration (left-handed with negated *V* component) gives the result more rigorously.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\left(-\frac{\pi}{2}\right) & -\sin\left(-\frac{\pi}{2}\right) \\ 0 & \sin\left(-\frac{\pi}{2}\right) & \cos\left(-\frac{\pi}{2}\right) \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} U \\ -V \\ W \end{pmatrix} = \begin{pmatrix} U \\ W \\ V \end{pmatrix}$$

***Summary**:*

Probe used on a moving platform (e.g. aircraft, drone, car etc.):

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \|\mathbf{U}\| \begin{pmatrix} \cos(\beta)\cos(\alpha) \\ \sin(\beta)\cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$$

Probe used in a wind tunnel with right-handed coordinate system (*z*-axis vertical), probe pointing upstream:

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \|\mathbf{U}\| \begin{pmatrix} \cos(\beta)\cos(\alpha) \\ -\sin(\beta)\cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$$

Probe used in a wind tunnel with rotated right-handed coordinate system (*y*-axis vertical), probe pointing upstream:

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \|\mathbf{U}\| \begin{pmatrix} \cos(\beta)\cos(\alpha) \\ \sin(\alpha) \\ \sin(\beta)\cos(\alpha) \end{pmatrix}$$

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